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A scheme for preparation of multi-atom entanglement by detecting the cavity decay and analysis of its implementation

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We give the detailed study of a scheme to efficiently engineer multi-atom entanglement by detecting the cavity decay through single-photon detectors. The scheme can be used to prepare arbitrary superpositions of multi-atom Dicke states, without the requirements of high-efficiency detection, separate addressing of different atoms, and full localization of the atoms to the Lamb-Dicke limit. We analyze in detail various sources of noise and imperfections in this experimental scheme, and show that the scheme is robust to the dominant sources of noise and realizable with the state of the art technology.

Keywords: entanglement, cavity QED, Dicke state

I. INTRODUCTION

Engineering of quantum entanglement has various applications, ranging from fundamental tests of quantum mechanics [1], high-precision measurements [2], to implementation of quantum communication and computation [3]. Although quantum entanglement is typically fragile to practical noise and technical imperfections, there exist elegant ways to overcome this sensitivity by designing schemes with inherent robustness to diverse sources of noise. Some schemes with this property have been known for entangling two single atoms [4–9] as well as for entangling macroscopic atomic ensembles [10,11]. In these schemes, one has feedback from some measurements. The schemes are typically probabilistic, i.e.. they succeed conditional on certain measurement results. The practical noise in the scheme only decreases the success probability, but has no influence on the fidelity when the scheme succeeds. In this way, a high-fidelity entangled state can be obtained simply by repeating the scheme more times.

Recently, we have proposed a robust scheme to produce and engineer entanglement between multiple atoms in optical cavities [12]. Compared with the previous schemes [4–11], this scheme has the following favorable features. *(i)* It is much more efficient in the sense that the success probability can be close to unity, whereas in the previous schemes [4,5,8–11], the success probability is required to be much smaller than 1 to have the property of inherent robustness. *(ii)* It is more insensitive to certain practical sources of noise, such as randomness in the atom's position, atomic spontaneous emission, and detector inefficiency. *(iii)* Individual addressing of atoms is not required [6], nor are single photon states as initial resources [7].*(iv)* Most importantly, this scheme is not limited to generation of two-atom entanglement. Based on current experimental technology, we show that it is possible to generate any superposition of the Dicke states [13] between multiple atoms in an optical cavity. Superpositions of the Dicke states include the multi-party GHZ states their special cases, and useful for many applications in quantum information science [2,14,15]

Here, we first briefly review this scheme, and then give detailed analyses of its experimental realization. In particular, we analyze the influence of various sources of noise and imperfections in the current experimental setups [16–18], and show that this scheme is still workable with the presence of noise.

II. THE ENTANGLING SCHEMES FOR TWO AND MULTIPLE ATOMS

To explain the scheme, let us start from the simplest case with two atoms trapped in two different cavities. The schematic setup is shown in Fig. 1A, with the relevant atomic levels depicted in Fig. 1B. The states $|g\rangle$, $|0\rangle$, $|1\rangle$ correspond to the hyperfine and the Zeeman sublevels of alkali atoms in the ground-state manifold, and $|e\rangle$ corresponds to an excited state. The atom is initially prepared in the state $|g\rangle$, but the basis-vectors of a qubit are represented by the states $|0\rangle$ and $|1\rangle$. The transition $|g\rangle \rightarrow |e\rangle$ is driven adiabatically through a classical

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laser pulse with the corresponding Rabi frequency denoted by $\Omega(t)$ [19]. With the driving pulse, the atom is transferred with probability $p_c \simeq 1$ to the $|0\rangle$ and $|1\rangle$ states by emitting a photon from the transitions $|e\rangle \rightarrow |0\rangle$ or $|e\rangle \rightarrow |1\rangle$. Without loss of generality, we assume that the transitions $|e\rangle \rightarrow |0\rangle$ and $|e\rangle \rightarrow |1\rangle$ are coupled to two degenerate cavity modes a^h and a^v with different polarizations h and v. The decay pulses from the two cavities are interfered at a polarization beam splitter (PBS), with the outputs detected by two single-photon detectors after a 45^o polarizer (denoted as P_{45} in Fig. 1A). For the decay pulse from the **R** cavity, a polarization rotator $R(\pi/2)$ is inserted before the PBS which exchanges h and v polarizations of the incoming photon. Conditioned upon registering one photon from *each* of the detectors, the two atoms in the cavities **L** and **R** will be prepared into the maximally entangled state

$$|\Psi_{LR}\rangle = (|01\rangle_{LR} + |10\rangle_{LR})/\sqrt{2}. \tag{1}$$

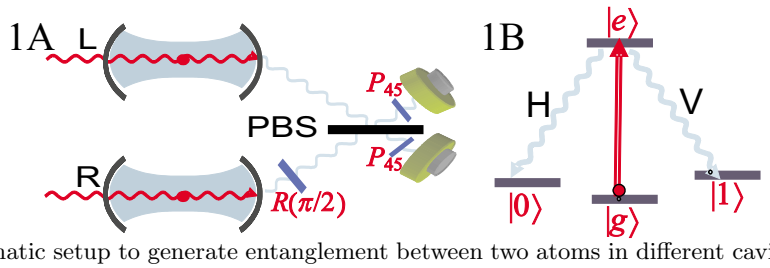

FIG. 1. 1a. The schematic setup to generate entanglement between two atoms in different cavities **L** and **R**. (2b). The relevant atomic level structure and the laser configuration.

To see this, we write down the interaction Hamiltonian in the rotating frame, which, for each of the cavities, has the form (setting $\hbar = 1$)

$$H = \Omega(t)\,|e\rangle\langle g| + g_0\,|e\rangle\langle 0|\,a^h + g_1\,|e\rangle\langle 1|\,a^v + H.c., \tag{2}$$

where g_0 and g_1 are the corresponding coupling rates. The cavity outputs a^μ_{out} $(\mu = h, v)$ are connected with the cavity modes a^μ through the standard input-output relations $\dot{a^\mu} = -i\,[a^\mu, H] - \kappa a^\mu/2 - \sqrt{\kappa}a^\mu_{in}(t)$ and $a^\mu_{out}(t) = a^\mu_{in}(t) + \sqrt{\kappa}a^\mu$ [20], where κ is the cavity decay rate, and $a^\mu_{in}(t)$, with the commutation relation $\left[a^\mu_{in}(t), a^{\mu\dagger}_{in}(t')\right] = \delta(t-t')$, denotes the vacuum cavity input. We are interested in the limit for which the variation rate of $\Omega(t)$ is significantly smaller than the cavity decay rate κ. In this limit, we can define an effective single-mode bosonic operator a^μ_{eff} from the cavity output operator $a^\mu_{out}(t)$ as $a^\mu_{eff} = \int_0^T f(t)\,a^\mu_{out}(t)\,dt$ (see Refs. [21,19]), where T is the pulse duration and $f(t)$ is the output pulse shape, which is determined by the shape of $\Omega(t)$ as $f(t) = \sqrt{\kappa}\sin\theta(t)\exp\left[-(\kappa/2)\int_0^t \sin^2\theta(\tau)\,d\tau\right]$ with $\sin\theta(t) = \Omega(t)/\sqrt{|g_0|^2 + |g_1|^2 + |\Omega(t)|^2}$. After the driving pulse, for each of the cavities λ (λ =**L**, **R**), the final state between the atom and the corresponding cavity output has the form

$$|\Psi\rangle_\lambda = (g_0\,|0\rangle_\lambda\,|h\rangle_\lambda + g_1\,|1\rangle_\lambda\,|v\rangle_\lambda)/\sqrt{|g_0|^2 + |g_1|^2}, \tag{3}$$

where $|\mu\rangle = a^{\mu\dagger}_{eff}\,|vac\rangle$, $(\mu = h, v)$, and $|vac\rangle$ denotes the vacuum state of the optical modes.

If the driving pulses have the same shape $\Omega(t)$ for the **L** and **R** cavities, the output single-photon pulses from the two cavities will also have the same shape $f(t)$, and they will interfere with high visibility at the polarization beam splitter (PBS). If one gets a "click" from each of the detectors at the outputs of the PBS, the two incoming photons can be either both in h polarizations or both in v polarizations, and these two possibility amplitudes are coherently superposed when the incoming photon pulses overlap with each other with the same shape. Therefore, the measurement in Fig. 1A, together with the polarization rotator $R(\pi/2)$, corresponds to projecting the whole state $|\Psi\rangle_L \otimes |\Psi\rangle_R$ between the atoms and the photons onto a subspace with the projection operator given by $P_s = |hv\rangle_{LR}\langle hv| + |vh\rangle_{LR}\langle vh|$. Within this measurement scheme, the state $|\Psi\rangle_L \otimes |\Psi\rangle_R$ is effectively equivalent to the four-particle GHZ state

$$|\Psi_{eff}\rangle \propto P_s |\Psi\rangle_L \otimes |\Psi\rangle_R \propto (|01\rangle_{LR} \otimes |hv\rangle_{LR} + |10\rangle_{LR} \otimes |vh\rangle_{LR}) / \sqrt{2}. \quad (4)$$

The 45^o polarizers in Fig. 1A project the photon polarizations to the $(|h\rangle + |v\rangle)/\sqrt{2}$ state. It immediately follows from Eq. (4) that after this measurement the two atoms will be prepared in the maximally entangled state (1). If one rotates the angles of the polarizers in Fig. 1A, corresponding a measurement of the incoming photon polarizations either in the $\{|h\rangle, |v\rangle\}$ basis or in the $\{(|h\rangle + |v\rangle)/\sqrt{2}, (|h\rangle - |v\rangle)/\sqrt{2}\}$ bases, one can further demonstrate four-particle GHZ-type of entanglement between the atoms and the photons as indicated by the effective state (4) [22]. The 45^o polarizer can also be replaced by a PBS with both of its outputs detected by single-photon detectors. The measurement success probability is then increased by a factor of 2 for each side, and the overall success probability of this scheme becomes $p_s = 2|g_0 g_1|^2 / \left(|g_0|^2 + |g_1|^2\right)^2$. This probability is about $p_s \sim 1/2$ in the case with $g_0 \sim g_1$, which shows that the present scheme is significantly more efficient than the previous schemes [4,5,8–11], where the success probability is required to be much smaller than 1 even in the ideal case.

We next extend our basic scheme to entangle multiple atoms in the same optical cavity. The schematic setup is shown by Fig. 2, with each of the N_a atoms taken to have the same level structure as depicted in Fig. 1B and with the atoms not separately addressable [16–18]. The initial state of the system has the form $|G\rangle = \bigotimes_{i=1}^{N_a} |g\rangle_i$ with all the atoms prepared to the ancillary state $|g\rangle$. The driving laser, incident from one side mirror, is now divided into M sequential pulses, with $M \geq N_a/2$. We assume that the intensity of the pulse is controlled so that for each of the M pulses, an approximate fraction $1/M$ of the atomic population is transferred adiabatically from the $|g\rangle$ state to the $|0\rangle$ or $|1\rangle$ states, by emitting on average N_a/M photons with h or v polarizations. The output photons from the cavity decay are split by a PBS according to their polarizations, and then registered through two single photon detectors (called h and v detectors, respectively). For each driving pulse, we may or may not get a click from the h or v detectors, which are assumed *not* to distinguish one or more photons. For the whole M pulses, we can count the total number of "clicks" (n_h, n_v) registered from the (h, v) detectors, respectively. Of course, $n_h + n_v \leq N_a$ since there are only N_a atoms. If it turns out that $n_h + n_v = N_a$, the following Dicke state results for the N_a atoms:

$$|N_a, n_h\rangle = c(n_h) \left(s_0^\dagger\right)^{n_h} \left(s_1^\dagger\right)^{N_a - n_h} |G\rangle . \quad (5)$$

Here, the collective operators $s_\mu^\dagger$ $(\mu = 0, 1)$ are defined as $s_\mu^\dagger = \sum_{i=1}^{N_a} |\mu\rangle_i \langle g|$, and the normalization coefficient $c(n_h) = [N_a! n_h! (N_a - n_h)!]^{-1/2}$. Except the trivial cases with $n_h = 0, N_a$, clearly the Dicke state $|N_a, n_h\rangle$ is entangled. The multi-atom Dicke states and the GHZ states in general belong to different classes of entangled states, and the Dicke states are relatively more robust to the influence of noise [14]. The Dicke states have some interesting applications in quantum information processing and in high precision measurements [15].

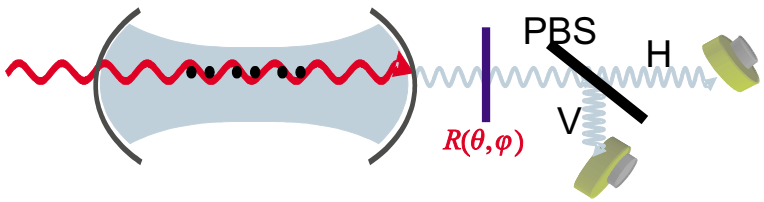

FIG. 2. 1a. The schematic setup to generate entanglement between multiple atoms in the same cavity. The polarization rotator $R(\theta, \varphi)$ is only required for generation of superpositions of the Dicke states.

To understand why a Dicke state results conditioned upon the above type of measurement, we note that each atom has an equal probability to emit a photon with the same pulse shape for each driving pulse for the assumed sequence of adiabatic passages. Hence, each driving pulse involves a collective excitation of the atoms to the $|0\rangle$ or $|1\rangle$ levels with homogeneous superposition coefficients. For the subset of measurements for which we register N_a photoelectric events in total from the h and v detectors for the whole M pulses, each "click" of the detectors should correspond exactly to the emission of one and only one photon by the atoms. This is the case

even if there are photon loss and detector inefficiencies, because we post select only the trials with exactly N_a photoelectric events. Therefore, for each "click" registered from the h or v detectors for these trials, we should apply correspondingly the collective operators $s_0^\dagger$ or $s_1^\dagger$ to the atomic state. After registering n_h h-polarized photons and $(N_a - n_h)$ v-polarized photons, we get exactly the state of Eq. (5).

Finally, we extend further the above scheme to generate any superposition of the Dicke states $|N_a, n_h\rangle$. For this purpose, we simply insert a polarization rotator $R(\theta, \varphi)$ before the PBS as shown in Fig. 2, which transforms the photon polarizations according to $|h\rangle \rightarrow \cos\theta\,|h\rangle + \sin\theta e^{i\varphi}\,|v\rangle$ and $|v\rangle \rightarrow -\sin\theta e^{-i\varphi}\,|h\rangle + \cos\theta\,|v\rangle$. We assume that the parameters θ, φ can be separately controlled for each driving pulse, and are denoted by θ_m, φ_m for the mth pulse. As before, we consider only the subset of cases for which exactly N_a photoelectric events are registered from the whole M-pulse sequence. If the h (or v) detector "clicks" for the mth pulse with the control parameters θ_m, φ_m, the corresponding atomic excitation operator P_{m0} (or P_{m1}) is expressed by the collective operators $s_\mu^\dagger$ as $P_{m\mu} = \cos\theta_m s_\mu^\dagger - (-1)^\mu \sin\theta_m e^{i\varphi_m(-1)^\mu} s_{1-\mu}^\dagger$ $(\mu = 0, 1)$. So, after N_a registered events, the final atomic state has the form $|\Psi_F\rangle = \prod_{i=1}^{N_a} P_{m_i\mu}\,|G\rangle$, where m_i $(i = 1, 2, \cdots, N_a)$ denote the set of driving pulses for which we register a photon. Each operator $P_{m_i\mu}$ introduces two real parameters $\theta_{m_i}, \varphi_{m_i}$, so there are $2N_a$ independently controllable real parameters in the state $|\Psi_F\rangle$. The state $|\Psi_F\rangle$ can be written in general in the form

$$|\Psi_F\rangle = \sum_{n_h=0}^{N_a} b(n_h)\,|N_a, n_h\rangle\,, \tag{6}$$

where the Dicke states $|N_a, n_h\rangle$ are defined by Eq. (5), and the complex superposition coefficients $b(n_h)$ are functions of $\theta_{m_i}, \varphi_{m_i}$. Superpositions of the Dicke states have $2N_a$ degrees of freedom, which exactly equals to the number of control parameters $\theta_{m_i}, \varphi_{m_i}$.

Actually, we can prove that an *arbitrary superposition* of the Dicke states $|N_a, n_h\rangle$ (i.e., the state $|\Psi_F\rangle$ with any coefficients $b(n_h)$) is obtainable by choosing an appropriate set of control parameters $\theta_{m_i}, \varphi_{m_i}$. For the proof, we write the state (6) in the form $|\Psi_F\rangle = b(N_a)\,c(N_a)\sum_{n_h=0}^{N_a} b'(n_h)\left(s_0^\dagger\right)^{n_h}\left(s_1^\dagger\right)^{N_a-n_h}|G\rangle$, where $b'(n_h) = c(n_h)\,b(n_h)\,/\,[b(N_a)\,c(N_a)]$, and without loss of generality we have assumed $b(N_a) \neq 0$. Each of the atomic excitation operators $P_{m_i\mu}$ can be expressed as $P_{m_i\mu} \propto \left(s_0^\dagger - r_{m_i\mu}s_1^\dagger\right)$, where the complex coefficient $r_{m_i\mu}$, determined by the real parameters $\theta_{m_i}, \varphi_{m_i}$, is the relevant control parameter. To prepare a desired state $|\Psi_F\rangle$ with the superposition coefficients $b'(n_h)$, we need to choose the parameters $r_{m_i\mu}$ to satisfy the algebraic equation $\prod_{i=1}^{N_a}\left(s_0^\dagger - r_{m_i\mu}s_1^\dagger\right) = \sum_{n_h=0}^{N_a} b'(n_h)\left(s_0^\dagger\right)^{n_h}\left(s_1^\dagger\right)^{N_a-n_h}$. It immediately follows from this equation that the parameters $r_{m_i\mu}$ should be the N_a solutions of the N_ath-order algebraic equation $\sum_{n_h=0}^{N_a} b'(n_h)\,x^{n_h} = 0$, where x denotes the variable. In the complex domain, there always exist N_a solutions to the N_ath-order algebraic equation, and the parameters $r_{m_i\mu}$ are uniquely determined from these solutions if we do not care about the order of the excitation operators $P_{m_i\mu}$ (note that they commute with each other).

This proves constructively that we can generate any superposition of the Dicke states by choosing appropriate parameters $\theta_{m_i}, \varphi_{m_i}$. Of course, to prepare such a superposition, the success probability of the scheme is typically significantly smaller than that for preparation of a Dicke state. However, for a few atoms, it is still reasonable to employ this scheme to prepare arbitrary superpositions of the states $|N_a, n_h\rangle$. Eq. (6) represents the complete set of states in the symmetric subspace for all the atoms, and is the largest set that can be prepared without separate addressing of different atoms. The symmetric states $|\Psi_F\rangle$ include many interesting states as their special cases, such as the well-known N_a-party GHZ state $|\Psi_{GHZ}\rangle = (|N_a, 0\rangle + |0, N_a\rangle)\,/\sqrt{2}$, which is simply a superposition of two Dicke states.

III. DETAILED ANALYSES OF THE EXPERIMENTAL IMPLEMENTATION

In this section, we would like to first show how to implement this scheme based on the real level structure for alkali atoms, and then analyze the influence of noise and imperfections in the experimental setups. We also show how to control the pumping laser intensity in experiments

A. The level configuration

We take the Cesium atoms as an example to show how to realize the required level configuration. The quantization axis is chosen to be along the cavity axis. Initially, the atoms are prepared into the Zeeman sublevel $6S_{1/2}\ |F=4,m=0\rangle$ through optical pumping. This can be done, for instance, by shining two pumping lasers, one unpolarized and resonant with the transition $6S_{1/2}\ |F=3\rangle \rightarrow 6P_{3/2}\ |F'=4\rangle$, and the other π-polarized and resonant with the transition $6S_{1/2}\ |F=4\rangle \rightarrow 6P_{3/2}\ |F'=4\rangle$. Under these two lasers, the atom has a unique dark state $|D\rangle = 6S_{1/2}\ |F=4,m=0\rangle$ as the dipole transition $6S_{1/2}\ |F=4,m=0\rangle \rightarrow 6P_{3/2}\ |F'=4,m'=0\rangle$ is forbidden, and it will be pumped to this state $|D\rangle$ within a time comparable to the life time of its upper level. Then, we can adiabatically transfer the atom to the level $|g\rangle = 6S_{1/2}\ |F=3,m=-1\rangle$ by applying two Raman laser pulses respectively resonant with the transitions $|F=4,m=0\rangle \rightarrow |F'=4,m'=-1\rangle$ (σ^--polarized) and $|F=3,m=-1\rangle \rightarrow |F'=4,m'=-1\rangle$ (π-polarized). The classical laser pulse in the above entangling schemes can then be chosen as a σ^+-polarized laser collinear with the cavity axis which is applied to the transition from $|g\rangle = 6S_{1/2}\ |F=3,m=-1\rangle$ to $|e\rangle = 6P_{3/2}\ |F'=4,m'=0\rangle$. The cavity has two eigen-modes a^h and a^v degenerate in frequency and with h and v polarizations, respectively. The cavity length is tuned so that the eigen-frequency of a^h and a^v is resonant with the atomic transition frequency $6S_{1/2}\ |F=4\rangle \rightarrow 6P_{3/2}\ |F'=4\rangle$. The other transverse and longitude cavity modes are far-off-resonance (the detuning is about 10 to 10^4 GHz). The cavity modes a^h and a^v couple the atoms from the upper level $|e\rangle = 6P_{3/2}\ |F'=4,m'=0\rangle$ respectively to the two ground states $|0\rangle = 6S_{1/2}\ (|F=3,m=+1\rangle + |F=3,m=-1\rangle)/\sqrt{2}$ and $|1\rangle = 6S_{1/2}\ (|F=3,m=+1\rangle - |F=3,m=-1\rangle)/\sqrt{2}$. We then get exactly the interaction Hamiltonian (2), which is the basic for the above entangling schemes. In this level configuration, due to the symmetry, the coupling rates g_0 and g_1 in the Hamiltonian (2) are the same.

If the birefringence of the cavity mirror is small or compensated at the atomic transition frequency $6S_{1/2}\ |F=4\rangle \rightarrow 6P_{3/2}\ |F'=4\rangle$, we can also choose σ^+ and σ^- as the two eigen-polarizations. In this case, we are able to use various different level configurations. For instance, we can also take $|g\rangle = 6S_{1/2}\ |F=3,m=3\rangle$, $|e\rangle = 6P_{3/2}\ |F'=4,m=3\rangle$, $|0\rangle = 6S_{1/2}\ |F=4,m=2\rangle$, and $|1\rangle = 6S_{1/2}\ |F=4,m=4\rangle$. In this case, in the Hamiltonian (2) $g_0^2/g_1^2 = 7/4$, determined by the ratio of the corresponding Clebsch-Gordan coefficients. Note that in our schemes, it is not necessary to have $g_0 = g_1$. We still get the maximally entangled state (1) with equal superposition coefficient in the case of $g_0 \neq g_1$.

The above level configurations are by no means specific to the Cesium atoms (for which the nuclear spin $I = 7/2$). One can easily find the corresponding level configurations for the Rubidium or Sodium atoms (for which $I = 3/2$).

B. Randomness in the atom's position

In the current experimental setups with cavity QED [16,17], the atoms can not yet be fully localized. Specially, the atoms are not able to be localized to the Lamb-Dicke limit, meaning that the variation of the atom's position should be much smaller than the optical wave length. As the coupling rates between the atom and the cavity modes depend on the atom's position through the cavity mode function, the uncontrolled atom's position will cause random variation in the coupling rates, which is a typical source of noise for quantum information processing with the current cavity QED setups. For instance, in the setup without a far-off-resonant trapping (FORT) beam [17], the atom is not trapped, and its position, as well as the coupling rates, randomly vary in experiments from trial to trial. With a FORT beam, the atom is trapped but not yet cooled to the Lamb-Dicke region with the current experimental capability. As a result, the coupling rates randomly vary within a factor of 2 in typical experiments [16]. To overcome this noise, we can use the ideas proposed in Ref. [19] by using adiabatic passages with the pumping laser collinear with the cavity axis and interacting with the atom through off-resonant drive of the cavity mode. In this configuration, the Rabi frequency $\Omega(t)$ and the coupling rates g_0, g_1 depend on the atom's position through the same cavity mode function. The pulse shape $f(t)$, which is determined by the ratios $\Omega(t)/g_0$ and $\Omega(t)/g_1$, thus becomes independent of the random variation in the atom's position. In this way, the system dynamics and the resultant entangled state is insensitive to the noise in the atom's position.

If one uses a travelling-wave cavity instead of a standing-wave cavity, the atom's position only affects the common phase of the coupling rates g_0 and g_1, and in this case, a transverse pumping configuration also suffices

since the randomness in the common phase of g_0 and g_1 has no influence on the final entangled state $|\Psi_{LR}\rangle$.

With the driving laser collinear with the cavity axis, one needs to filter the driving light before coupling the quantum output to the single-photon detectors. At the first impression, it might seem to be very hard to filter the strong classical driving field to get the single-photon quantum output. But actually in the cavity case, the filtering of the collinear pumping light is not a problem. The filtering becomes much easier compared with the free-space case (see Ref. [10]) since most of the classical pumping light has been filtered already by the high-finesse cavity itself. To briefly explain this point, let us assume that the classical pumping field drives one cavity mode with a large detuning Δ (typically hyperfine splitting, about 9 GHz) from one side mirror of the cavity (say side 1), and the cavity has a transmission $t_2 >> t_1$ so that the quantum output dominantly emerges from the other side of the cavity (side 2). Most of the driving field will be reflected at the side 1 and will not be mixed with the quantum output, and only a very small portion of the driving field will go through the cavity to the side 2. This small portion is given by the factor of $(\kappa/\Delta)^2 < 10^{-6}$, where κ is the cavity decay rate (line width). This demonstrates that most of the pumping field has been filtered by the cavity itself. As a typical estimation, based on the numerical simulations in Ref. [19], the transmitted pumping field at the side 2 is only about five times stronger than the single-photon quantum field. It is straightforward to use frequency selection (such as an interferometer) to separate such a weak field, as one has already demonstrated in the laboratory.

C. The photon loss and the efficiency of the scheme

The dominant noise in our schemes are photon loss, which includes several contributions, such as the atomic spontaneous emission, the absorption and loss of the cavity mirror, the output coupling inefficiency due to the imperfect matching to the cavity mode function, an the inefficiency of the single-photon detectors. The atomic spontaneous emission means that the atoms are transferred to the states $|0\rangle$ or $|1\rangle$, but the accompanying photons go to the free space instead of the output mirror of the cavity, so it has the same effect as the other sources of photon loss. In our schemes, we require to register N_a photon clicks, which is the maximum number of photons that we could get from N_a atoms. So, if one photon is lost due to the above noise, surely we will not pass the test, and we simply repeat the scheme until we finally succeed. When we succeed, obviously we still get the desired entangled state, and thus the photon loss has no influence on the fidelity of our entangling schemes, though it indeed decreases the success probability of the experiments.

To calculate the success probability of our schemes in the presence of noise, we note that in the two atom scheme, the success probability p_s is simply reduced by a factor of η^2, where $1-\eta$ denotes the total loss for each photon. In the multi-atom entangling scheme, we note that the stepwise driving method described above is actually equivalent to the following one-step driving method: we transfer all the atomic population to the $|0\rangle$ and $|1\rangle$ levels with a single driving pulse, but both of the h and v polarized photons after the PBS need to be further split equally into M paths through a series of beam splitters, with separate photoelectric detection for each path. The state in Eq. (5) corresponds to the case when n_h h-detectors and $(N_a - n_h)$ v-detectors register a photoelectric event. When two or more photons go to the same path, the number of detector events is certainly less than N_a. So, for overall success with N_a events, we require that each photon follows a distinct path, for which the success probability is given by $p_{si} = (2M)!/\left[(2M-N_a)!\,(2M)^{N_a}\right]$ (in total there are $2M$ paths. For simplicity, we have assumed $g_0 \approx g_1$ so that one has equal probability to get h or v photons.) All photon loss processes simply contribute to an under-count probability $1-\eta$ for each photon. Hence, the success probability to generate one of the Dicke states of Eq. (5) is $p_{succ} = \eta^{N_a} p_{si}$, while the probability to obtain a specific Dicke state $|N_a, n_h\rangle$ is $p_{n_h} = p_{succ} 2^{-N_a} N_a!/\left[n_h!\,(N_a - n_h)!\right]$. Excluding the trivial cases with $n_h = 0, N_a$, we then find that the success probability to obtain an entangled state from this scheme is $p_{en} = p_{succ}\left(1 - 2^{-N_a+1}\right)$, which tends to unity in the case $2M >> N_a$ if we neglect contributions from photon loss (i.e., $\eta \to 1$). This scheme could thus be quite efficient. For instance, with $\eta = 0.70$ (0.20) and $M = 50$ (10) pulses, $p_{en} = 0.018$ $\left(1.9 \times 10^{-4}\right)$ for $N_a = 10$ (5) atoms, so that repeating this scheme on average $1/p_{en} \approx 56$ $\left(5.4 \times 10^3\right)$ times leads to a high-fidelity entangled state between 10 (5) atoms. In current experimental setups [16,17], the typical duration Δt of the adiabatic pulse is a few hundred nanoseconds, so that the total duration $(M/p_{en})\,\Delta t \simeq 1$ (20) ms for entangling 10 (5) atoms with $\eta = 0.70$ (0.20). The currently available trapping time of atoms in high-Q cavities is about 1 second [16].

D. Control of the pumping laser intensity

For the stepwise driving method, it is better to control the pumping laser intensity so that for each pulse about N_a/M atoms will be pumped to the $|0\rangle$ or $|1\rangle$ levels. Now we show how to control the pulse intensity to fulfill this requirement. For the mth pulse with an adiabatically varying Rabi frequency $\Omega_m(t)$, the transfer probability for each atom is given by [19]

$$p_m = 1 - \exp\left(-\kappa \int_0^T \Omega_m^2(\tau) \,/\, \left(|g_0|^2 + |g_1|^2 + |\Omega_m(\tau)|^2\right) d\tau\right). \tag{7}$$

We request that the average number of the emitted photons from the mth pulse $N_a p_m p_{m-1,g}$ approximately equals to N_a/M, where $p_{m-1,g}$ is the probability of the atom in the $|g\rangle$ level before the mth pulse, which is given by $p_{m,g} = p_{m-1,g}(1-p_m)$ and $p_{0,g} = 1$. From these relations, we get $p_m \approx 1/(M-m+1)$. For simplicity, one can assume that all the pumping pulses $\Omega_m(t)$ have the same shape but with different peak intensities Ω_{mp}. With the explicit form of p_m, one can solve from Eq. (7) the ratios between the different peak intensities Ω_{mp}.

IV. SUMMARY AND CONCLUSION

In summary, we have presented a method for efficient preparation and engineering of multi-atom entanglement in optical cavities. Firstly, we can generate entanglement between two atoms located in two distant cavities. This kind of non-local entanglement, together with the high detection efficiency available for atoms based on the quantum jump techniques [23], could be useful for Bell inequality detection, by closing both the detection efficiency and the non-locality loopholes. Though quite a lot of experiments have been reported for Bell inequality detection, none of them have closed both of the loopholes associated with the detection inefficiency and the nonlocality [23]. Secondly, we can also generate multi-atom entanglement inside an optical cavity without separate addressing of different atoms. This is an important feature of this scheme since it is still very hard to achieve separate addressing of different atoms in the same optical cavity with the currently available technology. The generated entanglement can be directly detected [24] and used for some applications, such as high-precision interferometers [2], without the requirement of separate addressing.

Our schemes well fit the status of the current experimental technology, as they are inherently robust to many sources of practical noise and technical imperfections, including the atomic spontaneous emission, various kinds of photon loss, and random variation in the atom's position. Our estimates suggest that it would be reasonable to exploit these ideas to generate high-fidelity entangled states for a sample of $N_a \sim 10$ atoms based on current capabilities in cavity QED [16], which represents a very exciting experimental possibility.

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